EX: continued fr. last day.

Find impulse responce

$$y[n]-\frac{1}{2}y[n-1]=x[n]-\frac{1}{4}x[n-1]$$

SOL:

Method 2

let $x[n]=\delta[n]$

then $y[n]=h[n]$

$$\therefore\ h[n]-\frac{1}{2}h[n-1]=\delta[n]-\frac{1}{4}\delta[n-1]$$

DTFT,

$$H(e^{jw})-\frac{1}{2}e^{-jw}H(e^{jw}) = 1-\frac{1}{4}e^{-jw}$$

$$H(e^{jw})=\frac{1-\frac{1}{4}e^{-jw}}{1-\frac{1}{2}e^{-jw}}$$

EX: find the responce of the following system

$$x[n]=\left(\frac{1}{2}\right)^n u[n]$$

$$y[n] - \frac{1}{2}y[n-1] = x[n] - \frac{1}{4}x[n-1]$$

$$x[n] = \delta[n]$$

$$y[n] = h[n]$$

$$H(e^{j\omega}) - \frac{1}{2}e^{-j\omega}H(e^{j\omega}) = 1 - \frac{1}{4}e^{-j\omega}$$

$$H(e^{j\omega}) = \frac{1 - 1/4\, e^{-j\omega}}{1 - 1/2\, e^{-j\omega}}$$

$$X(e^{j\omega}) = \frac{1}{1 - \frac{e^{-j\omega}}{2}}$$

$$Y(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega})$$

$$= \frac{1 - 1/4\, e^{-j\omega}}{\left(1 - \frac{e^{-j\omega}}{2}\right)\left(1 - (1/2)e^{-j\omega}\right)}$$

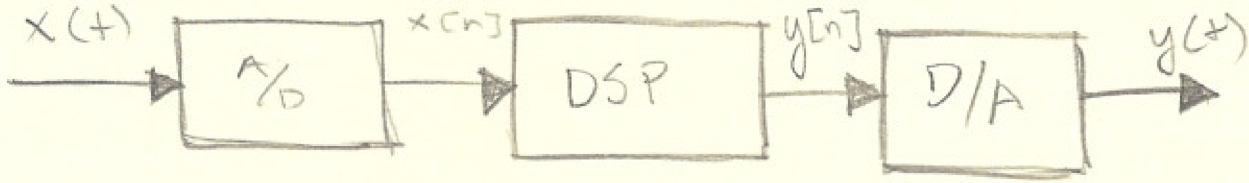
$$= \frac{1 - 1/4\, e^{-j\omega}}{\left(1 - \frac{e^{-j\omega}}{2}\right)^2}$$

from table

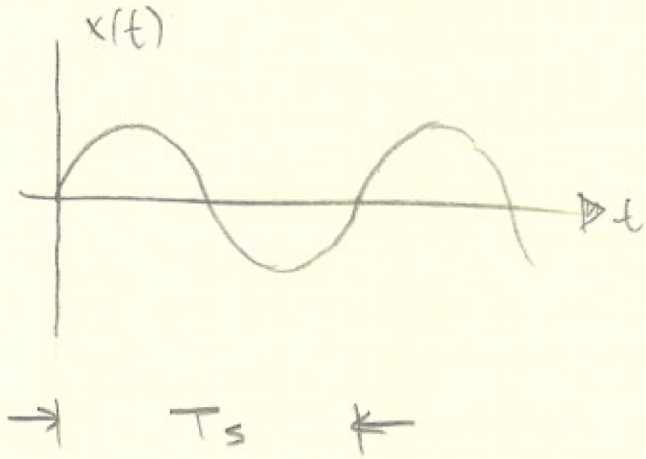
$$y[n] = (n+1)\left(\frac{1}{2}\right)^n u[n] - \frac{1}{4}(n)\left(\frac{1}{2}\right)^{n-1} u[n-1]$$

1.9 Sampling & reconstruction of Analog signal

Digital processing of an analog signal.



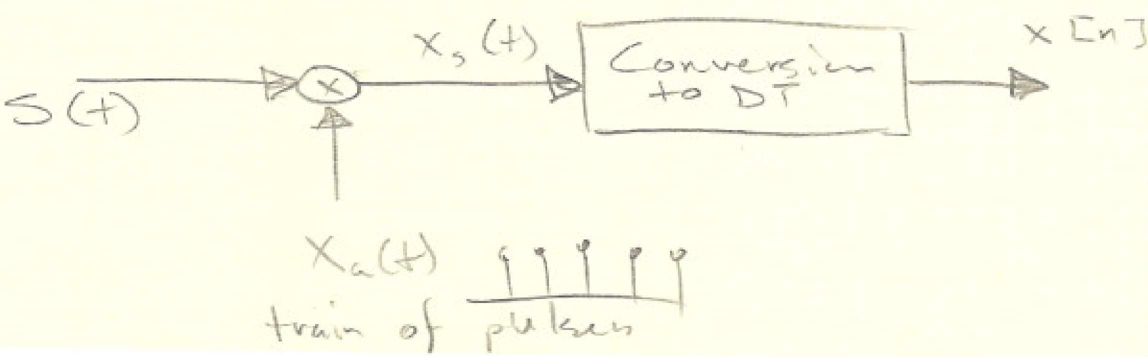
In order to reconstruct an analog signal we need to know how fast to sample.



1.9.1 frequeny domain repersentation of Sampling.

AKA: the relation of DTFT & FT.

Sam



x_a : analog signal

x_s : sampled version of analog signal

$x[n]$: discrete sequence

Ω : analog freq

ω : discrete freq

T_s : sampling time

$s(t)$: $\sum_{n=-\infty}^{\infty} \delta[t - nT_s]$; train of impulses.

$$x_s(t) = x_a(t)\, s(t)$$
$$= \sum_{n=-\infty}^{\infty} x_a(nT_s)\, \delta(t - nT_s)$$

Now,

$$S(j\Omega) = \mathcal{F}\{s(t)\} = \frac{2\pi}{T_s} \sum_{k=-\infty}^{\infty} \delta(\Omega - k\Omega_s)$$

$$X_s(j\Omega) = \mathcal{F}\{x_s(t)\}$$
$$= \mathcal{F}\{x_a(t)\, s(t)\}$$
$$= X_a(j\Omega) * S(j\Omega) \cdot \frac{1}{2\pi}$$
$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} s(\tau)\, x_a(j\Omega - \tau)\, d\tau$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{2\pi}{T_s} \sum_{k=-\infty}^{\infty} \delta\left(\frac{\tau}{j} - k\Omega_s\right) = 1$$

$\tau = kj\Omega_s$

?wtf?

$$\cdot\, x_a(j\Omega - \tau)\, d\tau$$

$$\boxed{\tau = j\Omega}$$

$$= \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X_a(j\Omega - jk\Omega_s)$$

$$X_s(j\Omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n T_s}$$

We know

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$X(e^{j\omega}) = X_s(j\Omega)\Big|_{\omega = \Omega T_s}$$

$$= \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X_a(j\Omega - jk\Omega_s)\Big|_{\omega = \Omega T_s}$$

$$\therefore\ X(e^{j\omega}) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X_a\left(\frac{j\omega}{T_s} - jk\frac{2\pi}{T_s}\right)$$

see chapter 4